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Evaluation of Discrete-Time Well-Conditioned State Observers

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Model-based monitoring systems based on state observer theory often have poor performance with respect to accuracy, bandwidth, reliability (false alarms), and robustness. Previous works have investigated quantitatively the above limitations from the machine monitoring viewpoint and have developed a design methodology for discrete-time well-conditioned state observers. In this paper, the estimation performance of well-conditioned observers is demonstrated on a DC spindle system designed and built for this purpose. The results show that the robustness of the estimate is similar to that obtained with the well-known Kalman filtering technique. Additional simulation-based examples show that the transient as well as steady-state error robustness to perturbations is better than or equal to Kalman filter performance depending on the nature of the modeling error. Because of this robustness, the well-conditioned observer for discrete-time systems is an important technique for the development of improved machine monitoring systems.

Key Words: Observer, Well-Conditioned, Experiment, Kalman Filter

1. Introduction

In many observer-based monitoring systems, the application of deterministic state observer techniques can be severely restricted because of limitations in accuracy, reliability and robustness. These observers can produce large transient or steady-state errors which can cause frequent false alarms. Huh and Stein (1994) identified ill-conditioning factors as the cause of the above phenomena and demonstrated that the effects of the factors on observer performance are governed by a unified main index, $\kappa_2(P)$ (condition number of the eigensystem P in terms of L_2 norm). In a subsequent paper (Huh and Stein, 1995), a design methodology for well-conditioned state observers was developed based on the unified main index. The above methodology allows a designer to select an observer gain to guarantee well-conditioned performance from the machine monitoring viewpoint. This is in contrast to classical observers (Luenberger, 1966) which can be designed based on pole-placement techniques. In another paper (Huh and Kim, 1995), the above methodology has been extended to the discrete-time case so that they can be easily implemented in real-time monitoring systems. As in the continuous-time case, a performance index is determined for discrete-time state observers and is shown to be expressed in an L_2-norm based condition number. Because the determined index is similar to the one selected in the continuous-time case, the design procedure for well-conditioned observers is similar to that developed in Huh and Stein (1995).

The performance of well-conditioned observers has been investigated in Huh and Stein (1994 and 1995) using numerical simulation. This paper investigates the performance of the well-conditioned observer on a laboratory-based DC spindle drive system designed and built for this purpose. In particular, the effects of model uncertainties on observer performance are compared to that obtained with a well-known Kalman filtering technique (Kalman, 1960). This is particular-

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ly important for state observers used in industrial environments, where significant noise and uncertainty can cause the observer to perform poorly as compared to assumptions about those uncertainties made in computer simulation experiments. In addition, another mechanical spindle system is represented in a computer simulation in order to compare the estimation robustness of the proposed observer to a Kalman filter.

2. Experimental Evaluation

2.1 DC-spindle drive set-up

A DC-spindle drive set-up is built to evaluate the performance of a well-conditioned observer compared with those of conventionally designed observers (e. g. Luenberger, 1966; and Kalman, 1960). The Experimental setup is shown in Fig. 1 The spindle drive consists of a DC-motor with a tachometer, a compliant shaft, bearings, and couplings. The motor is excited by the voltage produced from the linear amplifier and is connected to the drive train through the flexible coupling. The drive train consists of three bearings and a shaft with one section enlarged to increase its inertia (in order to increase the mechanical time constant). The armature voltage, motor speed and load speed are measured and digitized at 200 Hz. Assuming that the spindle drive parameters are time invariant, a 4th order linear state equation for the spindle drive system is derived. Based on the measured motor speed and the derived model, the load speed is estimated and verified by the measured load speed.

First, a continuous-time model for the above spindle drive is obtained based on the physical law. The parameters of the motor are obtained from the manufacturer's data sheet and the inertia of the drive train is calculated based on the dimensions of the system. Because the bearing friction parameter is installation dependent, it is obtained by steady-state power consumption experiments. The value of the Coulomb friction is tuned such that the simulated load speed matches the measured load speed. After substituting the numerical values on the model, the linear state equation is discretized using a zero-order-Hold equivalent sampler (0.005 second sampling time). Then a discrete plant model for the spindle drive is expressed as follows:

$$\begin{aligned} x(\kappa+1) &= \Phi x(\kappa) + \Gamma u(\kappa) \\ y(\kappa) &= Hx(\kappa) \end{aligned} \tag{1}$$

where

$$\Phi = \begin{bmatrix} 0.7993 & 0.0141 & 0.0022 & -0.0992 \\ -0.2622 & 0.2504 & -0.0123 & -0.0270 \\ 0.2003 & 0.0611 & 0.9958 & 0.1033 \\ 3.3077 & -0.0484 & -0.0374 & 0.7995 \end{bmatrix},$$

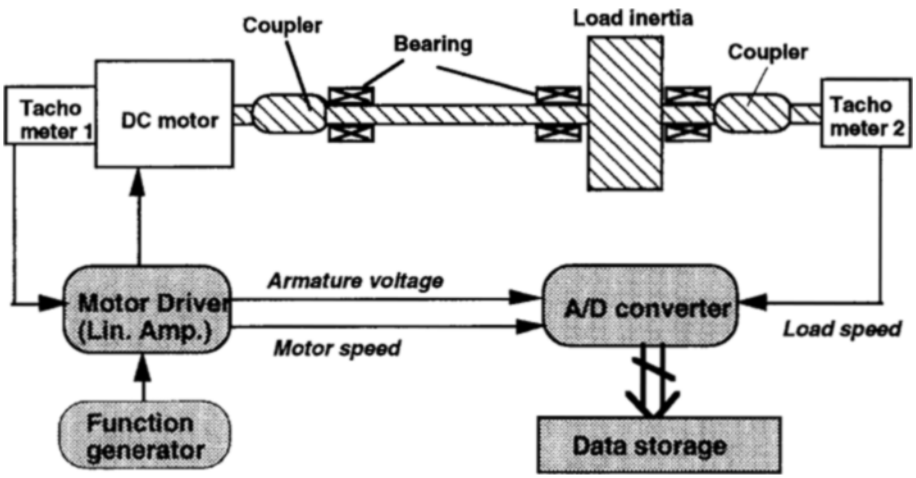

Fig. 1 Experimental setup

$$\Gamma = 10^{-3} \times \begin{bmatrix} -0.0036 & -0.4671 \\ 2.6840 & -0.0668 \\ 0.2331 & -4.5115 \\ 0.2576 & -0.8466 \end{bmatrix}, \quad H = [1\ 0\ 0\ 0],$$

x = [motor speed; armature current; load speed; shaft twist angle]^T

The objective is to design a discrete-time observer capable of estimating the load speed based on the measurement of motor voltage and speed. In this example three different observer techniques (Luenberger-type observer, well-conditioned observer, and Kalman filter) are applied and their performances are compared. The performance of the observers is investigated in terms of settling time, transient shape and robustness to modeling errors. The observer-based monitoring specifications are assumed as follows: settling time for the estimation convergence is at most 1 second; transient error should be small (at most 100 % in the case of zero initial condition); and estimation error should be minimized for a modeling error (damping coefficient change in this example). The experimental protocol is as follows. A programmed voltage was applied to the motor during the time, 0 sec $\leq t \leq$ 16 sec to bring the motor from an initial speed of 150 rpm to steady-state speed of 450 rpm. The motor voltage, speed and load speed were recorded. Then the damping coefficients associated with the bearings was changed by injecting oil into them and another data set was collected.

2.2 Discrete-time observer design

2.2.1 Luenberger-type observer design (Luenberger, 1966)

Based on the required settling time of 1 sec, a full-order discrete Luenberger observer can be built to estimate the unmeasurable states,

$$\hat{x}(\kappa+1) = \Phi\hat{x}(\kappa) + \Gamma u(\kappa) + L(y(\kappa) - H\hat{x}(\kappa)) \quad (2)$$

where $\hat{x} \in R^n$ is the estimate of the state variables. The observer is designed by a conventional pole-placement technique as follows:

Desired poles: $\Lambda = diag[0.97\ \ 0.94\ \ 0.9\ \ 0.87]$

Designed observer gain: $L = [-0.839\ \ 84.888\ \ -7.738\ \ 10.312]^T$

Designed observer eigenvalues:

$eig(\Phi - LH) = [0.97\ \ 0.94\ \ 0.9\ \ 0.87]$

The main index and the performance indices determined in Huh and Kim (1995) have the following values:

Main Index :

$\kappa_2(P) = 3.1588e+4$

Other Indices in Table-1 :

$\kappa_2(\Phi - LH - I) = 1.9703e+6$

$\kappa_1(P) = 2.5657e+4$

$1/|q_1^T p_1| = 1.9678e+3$

where $\kappa_2(P) = \|P\|_2 \|P^{-1}\|_2$ is the condition number for the eigenvector matrix P of the observer matrix $(\Phi - LH)$ in terms of an L_2 norm. $\kappa_2(\Phi - LH - I)$, $\kappa_1(P)$, and $|q_1^T p_1|$ are the condition number of the matrix $(\Phi - LH - I)$ in terms of the L_2 norm, the condition number of the eigenvector matrix P in terms of the L_1 norm, and the inner product of right and left eigenvectors, respectively.

Because the above Luenberger-type observer has very large values for performance indices, the performance of the designed observer is expected to be very ill-conditioned. The results obtained from estimating the load speed with a modeling error (injecting the lubrication oil in the bearings) and the initial conditions set to zero is illustrated in Fig. 2. This ill-conditioned observer gives a huge transient error, large transient-shape sensitivity to initial condition selection, and large steady-state estimation error due to the modeling

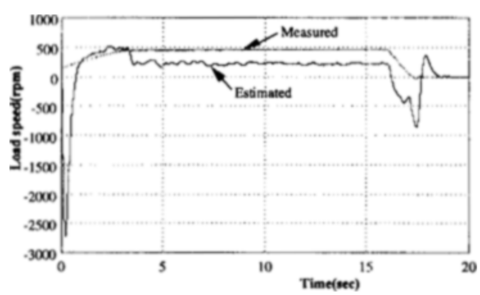

Fig. 2 Estimation of load speed from discrete-time Luenberger observer

error.

2.2.2 Well-conditioned observer design (proposed in this paper)

In the discretized plant model of Eq. (1), the system matrix Φ can be represented in block form.

$$\Phi=\begin{bmatrix} 0.7993 & 0.0141 & 0.0022 & -0.0992 \\ -0.2622 & 0.2504 & -0.0123 & -0.0270 \\ 0.2003 & 0.0611 & 0.9958 & 0.1033 \\ 3.3077 & -0.0484 & -0.0374 & 0.7955 \end{bmatrix}$$

According to the design procedure developed in Huh and Stein (1995), the above system matrix can be designed into a well-conditioned observer matrix by utilizing a scaling matrix and selecting a proper observer gain. The designed observer eigenvalues are separated as widely as possible inside the desired convergence region (Huh and Stein, 1995). The designed scaling matrix S, the observer gain L, and the observer matrix Φ'' are

$$S=diag[1\ 1\ 2.2244\ 1.3386],$$

$$L=\begin{bmatrix} l_1 \\ L_n-q \end{bmatrix}=\begin{bmatrix} 0.7634 \\ -0.2481 \\ 0.0949 \\ 2.338 \end{bmatrix},$$

$$\Phi''=\Phi'-LH$$

$$=\begin{bmatrix} 0.0359 & 0.0141 & 0.0048 & -0.1327 \\ -0.0141 & 0.2504 & -0.0274 & -0.0362 \\ -0.0048 & 0.0274 & 0.9958 & 0.0621 \\ 0.1327 & -0.0362 & -0.0621 & 0.7955 \end{bmatrix} \quad (3)$$

The above observer matrix has the following poles

$$eig(\Phi'')=[0.06\ 0.79\ 0.98\ 0.249],$$

and the values of the performance indices determined in Huh and Kim (1995) are:

Main Index :
$\kappa_2(P)=2.0730$
Other Indices in Table-1 :
$\kappa_2(\Phi'-LH-I)=47.6236$
$\kappa_1(P)=2.5986$
$1/|q_1^T p_1|=1.2106.$

Compared to the Luenberger-type observer, much smaller values of the performance indices are obtained. The transient and steady-state performance of the above observer is well-conditioned in the sense of eigenvalue sensitivity, transient error shape sensitivity, and modeling error robustness. The estimation performance result is shown in Fig. 3 using the same conditions as with the Luenberger-type observer. The estimate converges within the desired settling time with a negligible transient and steady-state error despite the presence of the modeling perturbation.

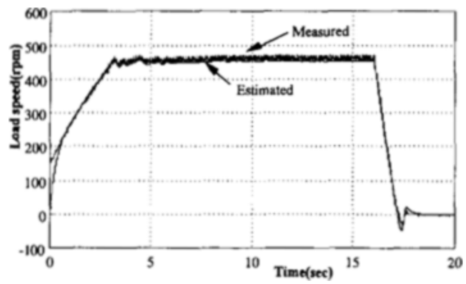

Fig. 3 Estimation of load speed from discrete-time, well-conditioned observer

2.2.3 Kalman filter design (Kalman, 1960)

In order to accurately judge the performance of the well-conditioned observer, the well-known and frequently utilized Kalman filter technique (Campbell et al, 1983; Broatch and Henley, 1991; and Grewal, 1986) is applied to this example problem. The development of a Kalman filter for this problem assumes a discrete-time system with state and measurement equations of the form

$$x(\kappa+1)=\Phi x(\kappa)+\Gamma u(\kappa)+Gw(\kappa)$$
$$y(\kappa)=Hx(\kappa)+v(\kappa) \quad (4)$$

where process and measurement noise have the following respective covariances:

$$E\{ww^T\}=Q\quad E\{vv^T\}=R\quad E\{wv^T\}=0,$$

A predictive form of the stationary Kalman filter can be designed to produce an LQG optimal estimate of states. The gain matrix L is determined from the algebraic Ricatti equation.

$$\bar{x}(\kappa+1)=(\Phi-\Phi LH)\bar{x}(\kappa)+\Gamma u(\kappa)+\Phi Ly(\kappa) \quad (5)$$

where $\bar{x}$ is expressed as the observation update and the state update equations.

$$\hat{x}(\kappa)=\bar{x}(\kappa)+L(y(\kappa)-H\bar{x}(\kappa))$$
$$\bar{x}(\kappa+1)=\Phi\hat{x}(\kappa)+\Gamma u(\kappa)$$

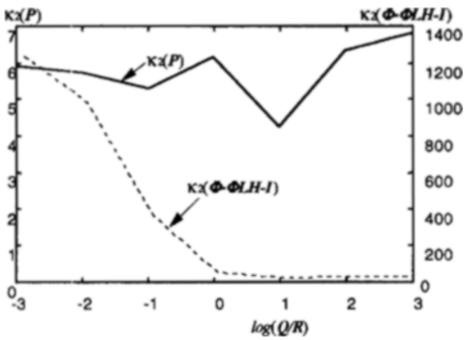

Fig. 4 Performance index values with respect to Q/R

The above Kalman filter algorithm of Eq. (5) is applied to this example with $G=[1\ 0\ 1\ 0]^T$. This corresponds to the time-varying damping coefficient parameters located in the first and third rows. The covariance matrices Q and R need to be determined from the stochastic characterization of the model and measurement. However, in order to make a conservative comparison to the well-conditioned observer, Q and R are tuned until the best estimation performance is obtained for the Kalman filter. This tuning procedure is also confirmed with the performance index values as shown in Fig. 4. The selected values for the covariance are $Q=0.011$ and $R=0.0011$.

The designed observer poles are

$$eig(\Phi-\Phi LH)=[0.1\ \ 0.24\ \ 0.915 \pm 0.041i]$$

with the performance indices (Huh and Kim, 1995) of

Main Index :

$\kappa_2(P)=4.2298$

Other Indices in Table-1 :

$\kappa_2(\Phi-\Phi LH\text{-}I)=52.5712$

$\kappa_1(P)=5.9666$

$1/|q_i^T p_1|=2.0369$

The Kalman filter estimate of the load speed compared to the measured speed is illustrated in Fig. 5 (The same modeling error as used for the Luenberger-type observer is used here). The estimation results look quite similar to the well-conditioned observer results. It shows that the steady-state estimation error $(\|x_3-\hat{x}_3\|_2/\|x_3\|_2 * 100\%)$ of the Kalman filter (1.1 %) is a little smaller than that of the well-conditioned observer (1.7 %), even though the Kalman filter has larger main performance index than the well-conditioned observer. This is not a contradiction, however, because the index is only related to the upper bound of the estimation error. The reader should be reminded that the Kalman filter is the best filter which was determined from a posteriori simulations. Filters with not so finely tuned

Q and R matrices can have larger steady-state errors and longer settling times. Therefore, the well-conditioned observer can be regarded as the best Kalman filter possible for the given estimation problem. This point is further illustrated in the following example.

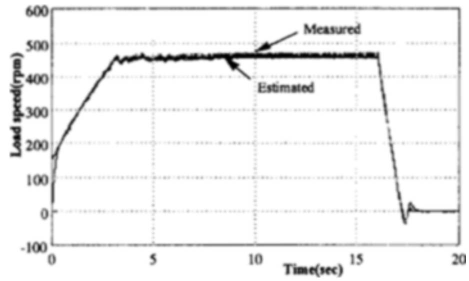

Fig. 5 Estimation of load speed from discrete-time Kalman filter

3. Simulation Example

Another spindle drive example is considered to further compare the performance of a well-conditioned observer with that of Kalman filter. A discrete plant model equation for spindle drive system is expressed as follows:

$$x(\kappa+1)=\Phi x(\kappa)+\Gamma u(\kappa)$$
$$y(\kappa)=Hx(\kappa) \tag{6}$$

where

$$\Phi=\begin{bmatrix} -0.0015 & 0.0001 & 0.0004 & 0.7366 & 0.0004 \\ -0.4304 & -0.0023 & 0.2363 & 419.9212 & 0.2520 \\ -0.0017 & 0.0001 & 0.0002 & 1.3064 & 0.0008 \\ 0 & 0 & 0 & -1.8 & -1 \\ 0 & 0 & 0 & 0.9 & 0 \end{bmatrix},$$

$$\Gamma=\begin{bmatrix} 0.000003 \\ 0.000001 \\ 0.000005 \\ 0 \\ 0.0000001 \end{bmatrix}$$

$$H=[1\ 0\ 0\ 0\ 0]$$

In this example, three cases of perturbations are considered: in the first case a plant perturbation exists in every row of the transition matrix Φ, in the second case there is a plant perturbation in the first, second and third rows of the transition matrix, and in the third case there is a 5 % measurement bias in output. The first two cases of plant perturbations are expressed as follows (The character $\otimes$ represents the perturbations):

Case 1:

$$\underline{\Phi}=\begin{bmatrix} -0.0015\otimes 2 & 0.0001 & 0.0004 & 0.7366 & 0.0004 \\ -0.4304 & -0.0023\otimes 2 & 0.2363 & 419.9212 & 0.2520 \\ -0.0017 & 0.0001 & 0.0002\otimes 2 & 1.3064 & 0.0008 \\ 0 & 0 & 0 & -1.8\otimes 0.9 & -1 \\ 0 & 0 & 0 & 0.9\otimes 0.9 & 0 \end{bmatrix}$$

Case 2:

$$\underline{\Phi}=\begin{bmatrix} -0.0015\otimes 2 & 0.0001 & 0.0004 & 0.7366\otimes 2 & 0.0004 \\ -0.4304 & -0.0023\otimes 2 & 0.2363 & 419.9212 & 0.2520 \\ -0.0017 & 0.0001\otimes 2 & 0.0002\otimes 2 & 1.3064 & 0.0008 \\ 0 & 0 & 0 & -1.8 & -1 \\ 0 & 0 & 0 & 0.9 & 0 \end{bmatrix}$$

3.1 Well-conditioned observer design

Similar to the experimental set-up, the system matrix Φ in Eq. (6) is represented in block form and is designed into a well-conditioned matrix form by scaling and selecting the observer gain.

3.2 Kalman filter design

The stationary Kalman filter algorithm of Eq. (5) is applied to this example. Plant perturbations are assumed to occur in every row of the discrete-time transition matrix and the matrix G is selected to be $G=[1\ 1\ 1\ 1\ 1]^T$.

The performance index values as well as the steady-state estimation errors of the two observers are compared in Table 1 and Table 2 for the two cases of plant perturbations. As shown in Table 1, the Kalman filter with $Q=1$ and $R=1000$ gives the best estimation robustness to the plant perturbation among the possible Kalman filters. Even though the Kalman filter has a large value for the performance index, it demonstrates perturbations robustness that is as good as the well-conditioned observer where the performance index value is much smaller. However, very different results are obtained for case 2. The results for the case 2 perturbation are shown in Table 2. The steady-state error performance between the best Kalman filter and the well-conditioned observer is different by several orders of magnitude. Moreover, Table 3 shows the third case where the best Kalman filter demonstrates very sensitive estimation error when there exists a 5 % output measurement bias. In contrast, the well-conditioned observer is much less sensitive to the measurement bias. These results indicate that the well-condi-

Table 1 Steady state error (%) of the well-conditioned observer and the Kalman Filter (Case 1 plant perturbation effect)

	Kalman Filter			Well-conditioned Observer
	$Q=1000,\ R=1$	$Q=1,\ R=1$	$Q=1,\ R=1000$	
$\kappa_2(\Phi-LH-I)$	8.2176×10^5	5.5631×10^5	7.0764×10^4	3.0357
$\kappa_2(P)$	2.4073×10^4	2.1264×10^4	3.3793×10^3	6.1623
$\lvert x_1-\hat{x}_1\rvert/x_1\times100$	0.5024 %	0.4227 %	0.2127 %	0.1371 %
$\lvert x_2-\hat{x}_2\rvert/x_2\times100$	53.3324 %	41.2246 %	8.8179 %	7.6758 %
$\lvert x_3-\hat{x}_3\rvert/x_3\times100$	0.3565 %	0.2713 %	0.0456 %	0.0378 %
$\lvert x_4-\hat{x}_4\rvert/x_4\times100$	44.3619 %	29.7841 %	6.3141 %	7.2973 %
$\lvert x_5-\hat{x}_5\rvert/x_5\times100$	13.5179 %	9.6869 %	0.5603 %	0.9284 %

Table 2 Steady state error (%) of the well-conditioned observer and the Kalman Filter (Case 2 plant perturbation effect)

	Kalman Filter			Well-conditioned Observer
	$Q=1000$, $R=1$	$Q=1$, $R=1$	$Q=1$, $R=1000$	
$\lvert x_1-\hat{x}_1\rvert/x_1\times100$	2.0997 %	1.7665 %	0.8889 %	0.5731 %
$\lvert x_2-\hat{x}_2\rvert/x_2\times100$	205.75 %	151.12 %	4.9231 %	0.229 %
$\lvert x_3-\hat{x}_3\rvert/x_3\times100$	1.324 %	0.9702 %	0.0331 %	8.8237×10^{-4} %
$\lvert x_4-\hat{x}_4\rvert/x_4\times100$	231.4 %	166.1 %	4.4038 %	1.2453×10^{-9} %
$\lvert x_5-\hat{x}_5\rvert/x_5\times100$	60.55 %	44.5 %	1.5430 %	4.011×10^{-10} %

Table 3 Steady state error (%) of the well-conditioned observer and the Kalman Filter (5 % Output measurement bias effect)

	Kalman Filter			Well-conditioned Observer
	$Q=1000$, $R=1$	$Q=1$, $R=1$	$Q=1$, $R=1000$	
$\lvert x_1-\hat{x}_1\rvert/x_1\times100$	7.2 %	5.291 %	0.1783 %	1.6615 %
$\lvert x_2-\hat{x}_2\rvert/x_2\times100$	1207 %	886.9 %	30.79 %	0.6141 %
$\lvert x_3-\hat{x}_3\rvert/x_3\times100$	7.8 %	5.7 %	0.1946 %	0.00523 %
$\lvert x_4-\hat{x}_4\rvert/x_4\times100$	1360 %	975.9 %	25.87 %	7.316×10^{-9} %
$\lvert x_5-\hat{x}_5\rvert/x_5\times100$	355.7 %	261.4 %	9.066 %	2.356×10^{-9} %

tioned observer has superior performance to a Kalman filter under the conditions tested and from the steady-state error viewpoint.

The most important implication that can be deduced from this simulation example is that good performance can be expected from systems with small performance index values. A filter (observer) with a large performance index does not necessarily mean large estimation error due to a plant perturbation or measurement bias. However, the filter will have large error for some perturbations and this depends on the orientation of the perturbation matrix, as investigated by Huh and Stein (1994). Thus, a filter can be considered as ill-conditioned in the sense that the estimation performance really depends on the environment and, therefore, good performance cannot be guaranteed. However, the well-conditioned observer proposed in this paper demonstrates directionally insensitive robustness with the limitation that it requires the conservative design condition derived by Huh and Stein (1995). This robustness property is very important to insure accurate, reliable monitoring for machine tool diagnostic systems.

4. Summary and Conclusion

The implementation of a well-conditioned observer has been demonstrated on a laboratory based spindle drive system. With the spindle-drive example, a Luenberger-type observer, Kalman filter, and a well-conditioned observer are designed, and their performances compared. The Luenberger-type observer shows severe sensitivity to modeling error. The Kalman filter gives satisfactory robustness to this same modeling error, but its performance is sensitive to the selection of covariance matrix and the orientation of the plant perturbation. Also, in selecting the Kalman filter gain, the time constant specification for the estimation convergence cannot be easily included at the design stage. In contrast, the well-conditioned observer allows for time constant specification at the design stage and demonstrates consistent transient shape, smaller estimation error due to modeling inaccuracy, and insensitivity to output measurement bias. This performance result is expected and guaranteed by a small value in the performance index. While further experiments are needed to consider the effects of noise, well-conditioned discrete-time state observers provide a promising technique for improving the accuracy and reliability of machine tool monitoring systems.

Acknowledgment

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